

Comparison between step strains and slow steady shear in a bubble raft

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We report on a comparison between stress relaxations after an applied step strain and stress relaxations during slow, continuous strain in a bubble raft. A bubble raft serves as a model two-dimensional foam and consists of a single layer of bubbles on a water surface. For both step strains and continuous strain, one observes periods of stress increase and decrease. Our focus is on the distribution of stress decreases, or stress drops. The work is motivated by apparent disagreements between quasistatic simulations of flowing foam and simulations of continuous strain for foam. Quasistatic simulations have reported larger average stress drops than the continuous strain case. Also, there is evidence in quasistatic simulations for a general divergence of the average size of the stress drops that only appears to occur in steady strain near special values of the foam density. In this work, applied step strains are used as an approximation to quasistatic simulations. We find general agreement in the dependence of the average stress drop on rate of strain, but we do not observe evidence for a divergence of the average stress drop.

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I. INTRODUCTION

An open question in the flow of foam is the correspondence between “true” quasistatic flow and constant rate of strain in the limit that the rate of strain approaches zero. (For reviews of foam and the flow behavior of foam, see for instance Refs. [1–3]). Experiments and simulations of model foams under constant rate of strain clearly exhibit limiting behavior in which the properties of the system become independent of the rate of strain for small enough rates of strain [4–9]. This has been referred to as the *quasistatic limit*. However, simulations have also been carried out in which a small step strain is applied to the system and the system is allowed to relax to a local energy minimum [10–12]. Such simulations are referred to as *quasistatic*. Surprisingly, results from quasistatic simulations and results from the quasistatic limit disagree with regard to certain aspects of the flow. This raises important questions not only for the flow behavior of foam, but also for a wide class of complex fluid materials, including granular systems, suspensions, colloids, and emulsions.

Understanding the quasistatic limit of complex fluids, along with glasses and supercooled liquids, is important in the context of the proposal that jamming provides a general theoretical framework in which to study these systems [13–15]. Jamming refers to the topological crowding of constituent particles, arresting their further exploration of phase space. The jamming phase diagram proposes the existence of a “jammed” state of matter as a function of temperature, stress and inverse density [13,15]. For materials with a yield stress, such as foam, there is an important connection between the jamming transition and the quasistatic limit. One definition of the yield stress is the value of stress below which a material behaves like an elastic solid and above which it exhibits “flow.” A careful treatment of the yield stress distinguishes between the transition to plastic deformation and plastic flow. But, for the purposes of this paper, one can treat the yield stress as the point at which the material “unjams.” For materials with a yield stress that are subjected

to a constant rate of strain in the quasistatic limit, the average stress is essentially the yield stress. Therefore, these systems exist in a state that is very close to the jamming transition. So, understanding the behavior of foam, or other materials, in the quasistatic limit is one way of probing the nature of the proposed jamming transition.

An open question for the jamming transition is whether or not it is a “true” phase transition. One feature of such transitions is the existence of divergences that exhibit well-defined scaling behavior. These issues have been explored in some detail for the case of zero stress as a function of density [16]. However, the question of the behavior at nonzero stress is still open. For foam, the issue of divergences and scaling behavior has been explored in some detail, even before the proposal of a jamming transition. This is particularly true for measurements of “avalanches” or stress drops in response to an applied strain. Under applied strain (whether continuous or step strain), foam initially responds in an elastic fashion. In this regime, the stress increases with strain. For sufficiently large applied strain, foam undergoes irregular periods of stress increase and decrease. Loosely speaking, a stress drop is a period of stress decrease, and stress drops are typically associated with nonlinear particle rearrangements. A long standing question in the study of foam is the nature of the distribution of stress drops and the length scales associated with regions of particle rearrangements.

Stress drops have been studied in a wide range of simulations, including the bubble model [4,5], the vertex model [17–19], the q -Potts model [20], and a quasistatic model [10,11]. (It should be noted that for periodic foams in two dimensions, analytic calculations of the stress under continuous shear have been carried out and interesting changes in the nature of the stress drops as a function of the fluid content are predicted [21]. However, these results are not directly applicable to the random systems discussed here.) Stress relaxations have been measured directly in experiments utilizing bubble rafts [8,9] and indirectly in other foam systems [6,22,23]. One question that can be asked is how does the distribution of stress drops depend on the density of

the system. In this regard, one can consider either the wet limit (increasing the fluid content until the foam “melts”) or the dry limit (the idealization of infinitely thin fluid walls). In the wet limit, a number of models show evidence for an increasing length scale [10,11,24], independent of whether or not the simulation models steady shear or step strains. In the other limit, evidence was found for diverging length scales [17–19], but there are still open questions as to the role of specific features of the model, such as having exactly zero fluid content.

If one considers the distribution of stress drops as a function of strain rate for foam that is not too close to either the wet or dry limit, then both simulations and experiment appear to divide into two categories. For the most part, simulations of a constant rate of strain, even in the quasistatic limit, report a distribution of stress drops that has a well-defined average value for all strain rates [4,5,24], with no evidence for a diverging length scale. In contrast, a number of quasistatic simulations [10–12] suggest that a diverging length scale does exist. For experiments that measure stress directly, only constant rate of strain experiments have been carried out in two dimensions, and these agree with the constant rate of strain simulations [8,9]. For experiments that only measure bubble rearrangements as an indirect measure of the stress drop distribution, continuous strain experiments show no evidence of a divergence in three dimensional [6] and two dimensional foams [22]. However, quasistatic measurements suggest the existence of large scale events [23]. These results raise two important questions. Is there a fundamental difference between quasistatic step strains and constant rate of strain? Or, is the difference in results simply a manifestation of differing definitions of stress drops?

This paper compares measurements of stress drops using two different types of applied strain. First, we reproduce earlier results for constant rate of strain experiments in bubble rafts [8,9]. Second, we study stress drops in response to applied step strains that are well separated by periods of waiting. With these experiments, we are able to compare the impact of various definitions of stress drops on measurements of the average stress drop. Also, we can compare experimental step strains with quasistatic simulations. As will be discussed in more detail, care is needed in making this comparison because there are qualitative differences between an experimental step strain and a true quasistatic step. However, a key feature of both that is *different* from continuous shear is the existence of a waiting period during which no *external* strain is applied to the system. Our results strongly suggest that this difference is important when considering the statistics of stress drops and by considering step strains, deeper insights into the fundamental “events” involved in stress releases are obtained.

II. EXPERIMENTAL SETUP

Our system is a two-dimensional foam system referred to as a bubble raft [25,26]. It consists of a single layer of gas bubbles floating on the surface of water, or other fluid. The Couette viscometer used to generate applied strain and measure the resultant stress is described in detail in Refs. [9,27].

The basic setup consists of two concentric “cylinders” that confine the bubbles in an annular region on the surface of water. The outer cylinder is a Teflon barrier composed of 12 segmented pieces. The barrier is able to compress and expand, so as to adjust the density of the bubble raft. For the experiments reported here, the radius of the outer barrier was varied between 6 cm and 11.5 cm. It is also able to rotate to generate either constant rates of strain or well-defined step strains. The inner barrier is suspended on a torsion wire and is free to rotate. The radius of the inner barrier was 2.7 cm. By measuring the rotation angle, the stress generated in the bubble raft is measured. The creation and characteristics of the bubble raft are discussed in detail in Ref. [8,9]. Essentially, nitrogen gas is bubbled through a solution of 80% water, 15% glycerine, and 5% Miracle Bubbles (Imperial Toy Corp.). The needle size and flow rate is adjusted to select the bubble distribution. A random size distribution of bubble radii ranging from 1 mm to 5 mm is used.

These experiments were all performed close to the time of creation of the bubble raft, within the first hour of creation when essentially no bubbles were observed to pop. An interesting feature of the bubble raft is that essentially no coarsening of the bubbles is observed during the duration of the experiments. This can be seen in measurements of the average stress during step strain experiments, which remains constant during the course of the experiment once the yield stress is reached. Because of this, we are unable to explore the impact of coarsening or creep on the stress relaxation. This should be contrasted with work in three-dimensional foams where creep flow prevents the system from maintaining a constant stress in response to a step strain over long time scales [28,29].

In this work, we focus on the nature of stress drops and the response to different types of strains applied. In addition, we consider the importance of the definition of a stress drop. To achieve these tasks, the focus is on step strain measurements. Step strains are generated by rotating the outer barrier at a relatively fast constant angular speed for a relatively short time period. Then, the outer barrier is held fixed for a selected time interval. This measurement is designed to parallel quasistatic simulations of foam in which the system is strained an increment and then energy is minimized. There are two aspects to a “true” quasistatic step that must be considered. First, the step itself should be small enough that it almost always produces a reversible deformation. Therefore, stress drops, associated with plastic deformations, should be rare. We will discuss the degree to which our system captures this feature later. Second, the system is relaxed until a minimum energy is found. In the experiments, we do not have access to direct measurements of the energy. So, we cannot determine at what point the energy of the system has achieved a minimum after the application of a step strain. Therefore, in order to facilitate comparison with theory, we systematically increased the waiting time until the results were independent of the waiting time. The expectation is that, at least in some statistical sense, this implies we are usually waiting until the energy is minimized.

There are three main variables of importance that define the step strains. They are the angular speed of the strain increment, Ω_{step} , the time for rotation, t_{rot} , and the time al-

lowed for relaxation, t_{rel} . One way to consider step strain measurements is to take the total angular displacement applied in a step and divide it by the time it took to strain it plus the time allowed for relaxation. This will provide an effective rotation rate,

$$\Omega_{\text{eff}} = \frac{\Omega_{\text{step}} t_{\text{rot}}}{t_{\text{rot}} + t_{\text{rel}}}. \quad (1)$$

It should be noted that because we are applying rapid, small strains, the bubble motions during the strain are essentially elastic. After the strain is stopped, the bubbles are either stationary or undergo nonlinear rearrangements. Therefore, the conversion of effective rotation rate to a rate of strain is not meaningful. In the case of constant applied rate of strain, the average bubble motions throughout most of the system are found to be consistent with various continuum models for fluids [30], so a definition of the rate of strain is possible. Therefore, for the purposes of this paper, the constant rate of strain results will also be reported in terms of the angular rotation rate, Ω , of the outer barrier. It is important to note that the rate of strain in the Couette geometry is a monotonic function of Ω [31]. Therefore, comparisons of effective rotation rate and actual rotation rate will provide insight into the connection between the quasistatic limit of constant strain rate and the step strain experiments. For a given effective rotation rate, we can probe different time scales and different dynamics by straining it for longer and longer times while allowing the system to relax by a proportionally increasingly long time. One question that can be considered is how an effective rotation rate compares to the actual rotation rate at that value. Are the stress drop distributions similar? Are the average stress values comparable?

It is important to take note of two distinct regimes of step strains. On one side are steps so small (much smaller than a particle diameter) that they are unlikely to independently induce a stress drop during the strain. The small step strains investigate the long time scale dynamics of infinitely slow strain rates, and approximate a quasistatic application of strain. On the other side is step strains that are large. In this case, the system is allowed to flow for some time, inducing several particle rearrangements. These experiments would look at the impact of flow induced rearrangements on the subsequent stress relaxation when flow is stopped. As the focus of this paper is comparison with the quasistatic limit of continuous rates of strain and with quasistatic simulations, we focus on the case of small step strains. In the following experiments, we selected a rotation of $\Omega_{\text{step}} = 0.01$ rad/s for 1 second. This choice was largely fixed by the physical limitations of the apparatus. However, it does correspond to a rate of strain on the order of 0.01 s^{-1} to 0.05 s^{-1} , which places it below the crossover to the quasistatic limit, $\dot{\gamma} \leq 0.07 \text{ s}^{-1}$ [9]. Also, the total angular displacement from such a step corresponds to displacements less than or equal to 1 mm, which is less than or equal to the smallest bubble radius.

III. RESULTS

Figure 1 shows a typical response from a series of small step strains. On the scale of the plot, each step strain corre-

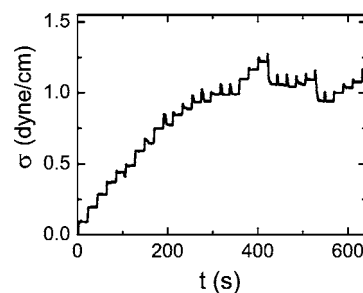


FIG. 1. A plot of the stress versus time for step strain measurements with a step time of 1 second at a rotation rate of 0.1 rad/s and a relaxation time of 20 seconds.

sponds to a sudden increase in the stress. The subsequent relaxation occurs during the waiting period. For the first couple of steps, the system is clearly behaving as an elastic solid, and the stress remains constant after a step strain. Furthermore, up until roughly 250 s, the average stress is increasing linearly with the applied strain. However, above approximately 0.5 dyne/cm, the individual steps begin to exhibit relaxation after the initial stress increase. Finally, the average value of the stress levels off after sufficient strain at a value of approximately 1 dyne/cm. These last two facts are consistent with observations from continuous rate of strain experiments that suggest a yield stress on the order of 0.8 dyne/cm [9,30]. Also, the agreement between the average stress for the step strains and the continuous rate of strain suggest that it is reasonable to compare the two types of flow.

Figure 2 illustrates a close up of multiple relaxation events that illustrate the range of responses to a step strain. The dotted lines mark the end of the step and the onset of a relaxation. Any given relaxation has two possible outcomes. First, the final stress can be greater than the stress *before* the application of the step strain. This is a stress increase. Or, the final stress can be lower than the stress before the application of the step strain. This is one definition of a *stress drop* for the quasistatic case. Independent of the final value of the stress, the relaxation process often occurs through multiple relaxations and plateaus indicative of the complex nature of the stress relaxation. The plateau regions presumably correspond to “quasibasins” during which the energy is still decreasing, but the decrease is releasing essentially no stress,

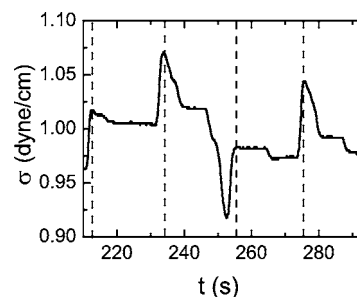


FIG. 2. A closer view of the stress vs time graph for step strains shown in Fig. 1. The dotted lines mark the end of the step and the onset of a relaxation. The multiple relaxations and plateaus that can be seen following a step strain indicate a complex relaxation time scale.

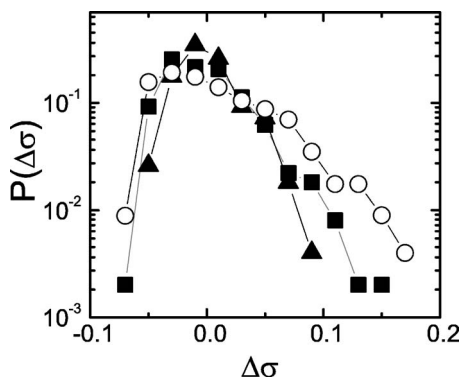


FIG. 3. A distribution of the change in stress defined as the difference between one step strain value and the next. Negative changes in stress correspond to stress increases while positive ones correspond to stress drops. The triangles are a waiting time of 1 second, the squares are 10 seconds and the circles are 20 seconds. As the waiting time is increased across this regime the stress distribution broadens and becomes more asymmetric.

until the system suddenly finds itself rapidly approaching a new value of stress. The fact that multiple plateaus occur complicates the determination of a true stress minimum during relaxation. This is one reason why we used multiple waiting times.

It should be noted that for waiting times greater than 10 s the relaxation plateaus for at least a few seconds before the next step is applied in over 90% of the steps. (The exact value varied from run to run, but for example, a typical run with a waiting time of 60 s had 46 events out of 50 total events plateau before the next step was applied.) For completeness, the events in Fig. 2 were selected to illustrate a rare event in which the stress had not plateaued before the next step, the step that occurs at approximately 250 s.

As mentioned, a stress drop is defined as the difference between one final value of stress just before a new step strain is made (i) and the next one ($i+1$). For purposes of comparing to previous work, we normalize the change in stress by the average stress for the given run ($\langle\sigma\rangle$):

$$\Delta\sigma = (\sigma_i - \sigma_{i+1}) / \langle\sigma\rangle. \tag{2}$$

Because we are mainly interested in the stress drops, it should be noted that with this definition a stress *drop* is *positive*.

As described previously, we report results for an applied rotation during the step of $\Omega_{\text{step}} = 0.01$ rad/s for 1 s and vary the waiting time. Figure 3 is a plot of the probability distribution for the stress drops for three different waiting times [20 s (○), 10 s (■) and 1 s (▲)]. (Recall, negative stress drops are stress increases.) The important feature to note on the change in stress distribution plots is that the large stress drop tail increases for increasing waiting times up to the time scale of about 20 s. As we will show, for waiting times greater than 20 s, the tail of the stress drop distribution appears to be independent of the waiting time. Two other features of the distribution should be noted. First, the average of the change in stress (including drops and increases) is essentially zero. This is important because it implies that a steady

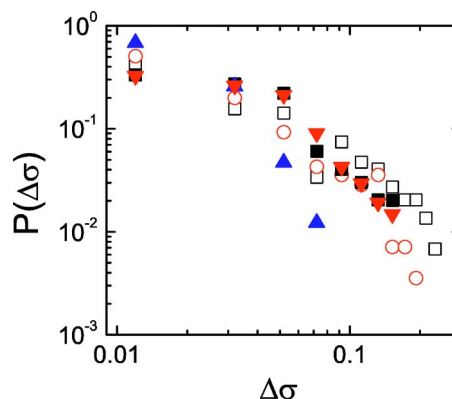


FIG. 4. Probability distribution for only the stress drops for both a series of step strains (closed symbols) and for continuous strain (open symbols). For long enough waiting times (>10 s) the overall shape of the distribution between the two types of flows are similar. The three waiting times for the step strain experiments are 1 s (▲), 20 s (■), and 60 s (▼). The two continuous rotation rates are $\Omega = 0.005$ rad/s (□) and $\Omega = 0.002$ rad/s (○). The results highlight the similarities of the step strain and continuous strain distributions for large stress drops.

state has been achieved. The distributions are also asymmetric, with a longer tail for the stress drops. Therefore, the most probable event is a stress increase.

Figure 4 is a plot of the probability distribution of only the stress drops and provides a comparison between steady rotation and step strains for one system size. The number of bubbles was 1.05×10^4 . It is important to realize that for continuous rate of strain, the definition of a stress drop is slightly different. In this case, because there is no well-defined waiting time, a stress drop is defined as *any* decrease in the stress. This definition was used in our previous measurements [8,9]. We will discuss the implications of this for the step strain experiments when we discuss the average stress drop size. The solid symbols are for the three different waiting times [1 s (▲), 20 s (■), and 60 s (▼)]. The open symbols are for the two different continuous rotation rates [$\Omega = 0.005$ rad/s (□) and $\Omega = 0.002$ rad/s (○)]. For long enough waiting times (>10 s), the distributions for continuous and step strain measurements both involve a clear cutoff at large stress drops, with no evidence for power law scaling in any of the distributions. Careful inspection of the distributions plotted in Fig. 4 reveals that during continuous strain there are significantly more small stress drops. This is not surprising given the two different operational definitions. For the step strain case, the entire relaxation is used, even if it is composed of multiple small steps. The degree to which the small stress drops dominate the continuous rate of strain is best illustrated by considering the average stress drop.

Figure 5 shows the average stress drop as a function of effective rotation rate for step strains (■) and actual rotation rate for the continuous rate of strain (▲) measurements. Here the dominance of the small stress drops is apparent. For the continuous strain case, we have reproduced the results reported in Ref. [9] that the average stress drop decreases with decreasing rotation rate. For the step strain case, we observe the behavior reported for simulations in Ref. [32] that the

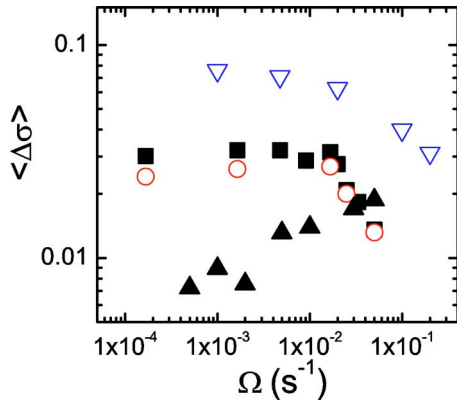


FIG. 5. Plot of the average stress drop as a function of Ω_{eff} for the step strain experiments (■) and as a function of Ω for the continuous strain experiments (▲). Also shown are the results for the step strain experiments with an alternative definition of a stress drop that counts each individual decrease during an entire relaxation event (○), and the alternative definition of stress drops for continuous strain, as discussed in the text (▽). The results highlight the differences between the two types of strain for the average stress drop measurements, as well as the impact of different definitions of a stress drop.

average stress drop increases with decreasing rate of strain and reaches a well-defined plateau.

To understand better the impact of the definition of the stress drop, we can plot two other quantities. First, for the step strain experiments, we can use the same definition as was used for the continuous rate of strain experiments, where any period of stress decrease is taken as a stress drop. This results in an increase in the number of small stress drops. The results for the average stress drop in this case are given by the open circles in Fig. 5. Here we see that this definition does decrease the average stress drop, but not to the degree that is observed in the continuous rate of strain case.

We also analyze the stress drops in the continuous rate of strain case by the method described in Ref. [32]. In this case, a stress drop is defined by taking an appropriate time interval, τ , and computing $\sigma(t) - \sigma(t + \tau)$. The time interval τ must be sufficiently large so as not to artificially break up a “typical” stress drop. This is achieved by measuring the average stress drop with increasing values of τ until the measurement is independent of τ . The average stress drop is then defined as $\langle \Delta\sigma \rangle = \langle \sigma(t) - \sigma(t + \tau) \rangle / \langle \sigma \rangle$. Two examples of the dependence of $\langle \Delta\sigma \rangle$ as a function of τ are illustrated in Fig. 6 for two rotation rates. A number of features of the behavior are interesting. First, the value of τ at which $\langle \Delta\sigma \rangle$ becomes independent of τ is an indication of the typical time over which an event occurs. One can see that this is of the order of 10 s in both cases. Second, the time appears to scale with rotation rate (or strain rate), suggesting that the events are best characterized by a typical strain interval.

The results for $\langle \Delta\sigma \rangle$ as measured by computing $\langle \sigma(t) - \sigma(t + \tau) \rangle$ are plotted in Fig. 5 (▽), as well. Here we again recover the behavior reported in Ref. [32] that the average stress drop increases with decreasing rate of strain. This result is not too surprising because this alternative definition deemphasizes small stress drops that occur on short time

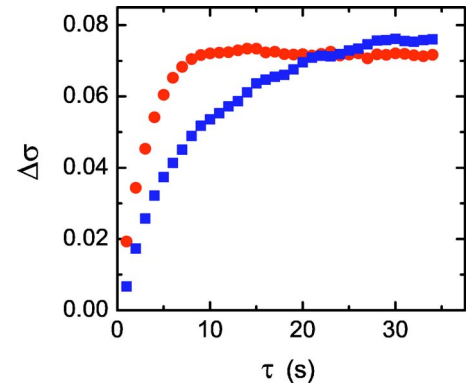


FIG. 6. Plot of $\langle \sigma(t) - \sigma(t + \tau) \rangle$ versus τ for two different rotation rates. The circles are $\Omega = 0.005$ rad/s, and the squares are $\Omega = 0.001$ rad/s. In both cases, the curves plateau for τ on the order of 10 s, with the exact value of the plateau proportional to the rotation rate.

scales. Also, this definition of the average stress drop is related to the variance of the stress as a function of time. The variance was reported in Ref. [9], and behavior similar to that reported in Ref. [32] was observed.

Finally, we considered the system size dependence of the average stress drop. This is illustrated in Fig. 7 for four different system sizes. The system sizes are given using two different measures, the average number of bubbles in the radial direction and the total number of bubbles. Also, we show data for two different effective rotation rates. The triangles correspond to data with a waiting time of 2 s and the squares correspond to data with a waiting time of 60 s. In all cases, there is no evidence of any system size dependence.

IV. DISCUSSION

In this study, we report on measurements of stress relaxations in response to small step strain increments followed

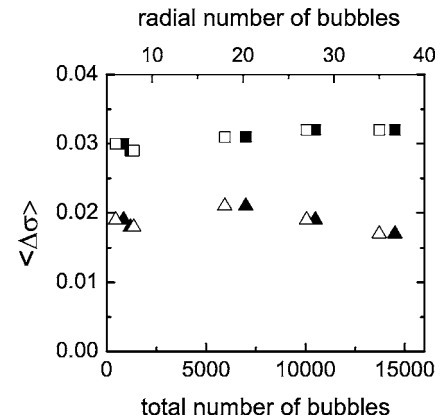


FIG. 7. Plotted here is the system size dependence of the average stress drop for two different waiting times. The squares are for a waiting time of 60 s, and the triangles are for a waiting time of 2 s. The open symbols correspond to the top axis, which gives the average number of bubbles in the radial direction. The closed symbols are relative to the bottom axis. In all cases, no observable system size dependence was measured.

by a fixed waiting time. These results are compared with similar measurements under the application of a constant rate of strain. The results show interesting differences between step strain measurements and constant rate of strain experiments. However, before discussing the comparison of the two experimental situations, it is worth commenting on the degree to which the step strains approximate quasistatic behavior as defined in simulations. Consideration of Figs. 2 and 5 provides insight into this question.

Recall, that there are two criteria for a quasistatic step, (1) the waiting time should be sufficiently long that the energy of the system is minimized and (2) the step should be sufficiently small that most of the time a stress drop is not induced. Figure 5 illustrates the degree to which condition (1) is met. By waiting sufficiently long, we find that the average properties are independent of the waiting time. This strongly suggests that at least statistically we are waiting long enough that the energy of the system can be considered to reach a minimum after each step. This is further confirmed by the fact that events which do not plateau at the end are extremely rare. In terms of condition (2), we see from Fig. 3 that the most probable event is a stress increase. However, stress increases represent only 57% of the events, not the 90+% of the events one would expect in a true quasistatic situation. Also, essentially all of the stress “increases” involve some relaxation of the stress generated during the applied step strain. Without measuring the details of bubble motion, it is difficult to determine the source of this relaxation. In particular, it would be interesting to know if nonlinear bubble rearrangements (such as T1 events) events occurred during these decreases or not. Independent of the source of this decrease, it is clear that condition (2) is at best approximately satisfied by having 57% of the events be stress increases. Therefore, it is important to be clear that the main results of this work involve comparisons between two well-defined experimental situations, step strains and continuous rotation. Any comparisons to quasistatic simulations need to be considered in the context of step strains that certainly involve a minimization of energy but may not be truly quasistatic because the strain involved is “relatively” large.

When comparing the step strains and continuous rotation, we find that the qualitative features of the two measurements are the same. The distribution of stress changes is asymmetric, with a tail for large stress drops. There is a well-defined average stress drop for all cases of interest, with no evidence for power-law scaling of stress drops for either continuous shear or step strains. However, quantitative comparison of the behavior of stress drops for the two cases yields some interesting differences (see Fig. 5). In order to clarify the discussion of Fig. 5, it is useful to clarify terms for each of the four different measurements of “stress drops.” For this discussion, we will use the term “stress event” to refer specifically to measurements of a decrease in stress from one maximum in stress to either the next minimum or plateau. This definition was applied to both the continuous rotation (\blacktriangle in Fig. 5) and the step strain (\circ) experiments. This term highlights that the measurements focus on simply the change in stress. We will introduce the term “energy event” to refer to the measurement of the *total* change in stress during a step strain (\blacksquare in Fig. 5). This designation highlights that these

stress drops typically correspond to a minimization of the energy during the event. Finally, we will use the term “continuous event” to refer to measurements of stress drops using $\langle\sigma(t) - \sigma(t + \tau)\rangle$ (∇ in Fig. 5). This term highlights that the measurement attempts to capture a typical “correlated event” during continuous rotation even when such events involve multiple individual stress drops.

Perhaps the most interesting result is that the average size of stress events decreases as a function of decreasing rotation rate for continuous rotation, but it increases as a function of decreasing rotation rate for step strains. This strongly suggests a fundamental difference between step strains measurement and continuous rotation, there are more small stress drops for slow, steady rotation than there are for step strains. The source of this difference is not obvious. One issue that future studies will explore is methods for detecting the equivalent of the quasibasins that occur after a step strain during continuous rotation. However, some insight into this difference is gained by considering the measurements of continuous events using $\langle\sigma(t) - \sigma(t + \tau)\rangle$, and the various time scales that appear to occur in the system.

From the step strain measurements, there is no change in the average stress drop for waiting times greater than ~ 10 s, suggesting a time scale for energy relaxation on this order. This roughly coincides with the time scale at which stress drops measured using $\langle\sigma(t) - \sigma(t + \tau)\rangle$ becomes independent of rotation rate (see Fig. 5). This suggests that the continuous events probed by $\langle\sigma(t) - \sigma(t + \tau)\rangle$ are related to the energy relaxation. But, the measurement of $\langle\sigma(t) - \sigma(t + \tau)\rangle$ provides a second time scale, the value of τ at which $\sigma(t) - \sigma(t + \tau)$ is independent of τ . From Fig. 6, our limited data suggests that τ is also of the order of 10 s for the rotation rates considered. But, it appears to scale with the rotation rate. This suggests that there is another “time scale” associated with the events, but this scale is set by a typical *strain* (on the order of 0.02). Combining this insight with the comparison of stress events suggests the following scenario for continuous rotation. As the rotation rate is decreased, one is able to resolve an increasing number of the small stress events that comprise a single continuous event. This exhibits as a measured decrease in the average stress drop, as determined by the stress event measurement, without a measurable change in $\langle\sigma(t) - \sigma(t + \tau)\rangle$, which is probing the entire “event” once τ is sufficiently long. In contrast, no such “increased resolution” occurs in the step strain measurements, because the same fixed strain is applied for every step. To probe the concept of a typical strain, future experiments will focus on the distribution of stress drops as a function of the strain amplitude for step strain measurements.

Finally, it is interesting to consider the issue of the “length” scale for the stress relaxation events. As mentioned in the introduction, there are two issues with regard to length scale, (1) as a function of the liquid fraction of the foam and (2) as a function of system size. Typically, one measures the length scale indirectly by measuring the average stress drop size or directly by measuring the number of particles involved in a stress drop and looking for system wide (or very large) events. Increases in length scale as a function of liquid fraction are observed in both continuous shear and quasi-

static strain for foams with sufficiently large liquid content, i.e., as one approaches the “melting” transition. Unfortunately, due to the nature of the bubble raft, this limit could not be explored in our experiments.

Scaling with system size has been observed in at least two different simulations of plastic-type flow. Of these, one is a simulation of the bubble model [12] and should be relevant to our experiment. The other one is a molecular dynamics simulation [32], and the connection to our experiment is not as direct. However, given the history of using bubble rafts to model crystalline and amorphous solids [25,26], it is worth considering the connection with a molecular dynamics simulation. For the range of system sizes studied here, we observed no dependence on the system size for either long or short waiting times in the step strain experiments. This was also the case for the average stress drops measured previously [9]. One possible explanation of the difference in scaling is the fact that the experiments are in a Couette geometry and the simulations use a square box. For our system, the system size was varied by increasing the radial dimension of the system. Though this does lead to a corresponding in-

crease in the azimuthal direction, the azimuthal direction remains periodic for all system sizes. The implication of this geometry is that bubble rearrangements in the azimuthal direction are not limited by any boundaries. As we are measuring the azimuthal stress, this may be the source of the size independence of the average stress drop. This possibility can be tested with simulations in a similar geometry. Also, experiments are planned to directly measure the spatial distribution of bubbles involved in the stress releases. Initial measurements of individual bubble motions were inconclusive with regard to the issue of the existence of system-wide events [33], so further work is required. Also, as noted in the introduction, experiments in three-dimensional foams provide strong evidence against system wide events [6].

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- [1] A. M. Kraynik, *Annu. Rev. Fluid Mech.* **20**, 325 (1988).
 [2] J. Stavans, *Rep. Prog. Phys.* **56**, 733 (1993).
 [3] D. Weaire and S. Hutzler, *The Physics of Foams* (Clarendon, Oxford, 1999).
 [4] D. J. Durian, *Phys. Rev. Lett.* **75**, 4780 (1995).
 [5] D. J. Durian, *Phys. Rev. E* **55**, 1739 (1997).
 [6] A. D. Gopal and D. J. Durian, *Phys. Rev. Lett.* **75**, 2610 (1995).
 [7] G. Debrégeas, H. Tabuteau, and J. M. di Meglio, *Phys. Rev. Lett.* **87**, 178305 (2001).
 [8] J. Lauridsen, M. Twardos, and M. Dennin, *Phys. Rev. Lett.* **89**, 098303 (2002).
 [9] E. Pratt and M. Dennin, *Phys. Rev. E* **67**, 051402 (2003).
 [10] D. Weaire, F. Bolton, T. Herdtle, and H. Aref, *Philos. Mag. Lett.* **66**, 293 (1992).
 [11] S. Hutzler, D. Weaire, and F. Bolton, *Philos. Mag. B* **71**, 277 (1995).
 [12] C. Maloney and A. Lemaitre, *Phys. Rev. Lett.* **93**, 016001 (2004).
 [13] A. J. Liu and S. R. Nagel, *Nature (London)* **396**, 21 (1998).
 [14] *Jamming and Rheology*, edited by A. J. Liu and S. R. Nagel (Taylor and Francis, London, 2001).
 [15] V. Trappe, V. Prasad, L. Cipelletti, P. N. Segre, and D. A. Weitz, *Nature (London)* **411**, 772 (2001).
 [16] C. S. O’Hern, L. E. Silbert, A. J. Liu, and S. R. Nagel, *Phys. Rev. E* **68**, 011306 (2003).
 [17] K. Kawasaki, T. Nagai, and K. Nakashima, *Philos. Mag. B* **60**, 399 (1989).
 [18] K. Kawasaki, T. Okuzono, T. Kawakatsu, and T. Nagai, in *Proceedings of the International Workshop of Physics of Pattern Formation*, edited by S. Kai (World Scientific, Singapore, 1992).
 [19] T. Okuzono and K. Kawasaki, *Phys. Rev. E* **51**, 1246 (1995).
 [20] Y. Jiang, P. J. Swart, A. Saxena, M. Asipauskas, and J. A. Glazier, *Phys. Rev. E* **59**, 5819 (1999).
 [21] D. A. Reinelt and A. M. Kraynik, *J. Fluid Mech.* **215**, 431 (1990).
 [22] M. Dennin and C. M. Knobler, *Phys. Rev. Lett.* **78**, 2485 (1997).
 [23] A. Abd el Kader and J. C. Earnshaw, *Phys. Rev. Lett.* **82**, 2610 (1999).
 [24] S. Tewari, D. Schiemann, D. J. Durian, C. M. Knobler, S. A. Langer, and A. J. Liu, *Phys. Rev. E* **60**, 4385 (1999).
 [25] L. Bragg and W. M. Lomer, *Proc. R. Soc. London, Ser. A* **196**, 171 (1949).
 [26] A. S. Argon and H. Y. Kuo, *Mater. Sci. Forum* **39**, 101 (1979).
 [27] R. S. Ghaskadvi and M. Dennin, *Rev. Sci. Instrum.* **69**, 3568 (1998).
 [28] A. D. Gopal and D. J. Durian, *Phys. Rev. Lett.* **91**, 188303 (2003).
 [29] S. Cohen-Addad, R. Hohler, and Y. Khidas, *Phys. Rev. Lett.* **93**, 028302 (2004).
 [30] J. Lauridsen, G. Chanan, and M. Dennin, *Phys. Rev. Lett.* **93**, 018303 (2004).
 [31] R. B. Bird, R. C. Armstrong, and O. Hassager, *Dynamics of Polymer Liquids* (Wiley, New York, 1977).
 [32] G. Picard, A. Ajdari, F. Lequeux, and L. Bocquet, *Phys. Rev. E* **71**, 010501(R) (2005).
 [33] M. Dennin, *Phys. Rev. E* **70**, 041406 (2004).